Memory-affecting Network Selection in Next Generation HetNets

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***Abstract*—In this paper, we study a long-run user-centric network selection problem in the 5G heterogeneous network, where the network selection strategies of the users can be investigated dynamically. Unlike the conventional studies on the long-run model, we incorporate the memory effect and consider the fact that the decision-making of the users is affected by their memory, i.e., their past service experience. Specifically, we model and study the interaction among the users in the framework of fractional evolutionary game based on the classical evolutionary game theory and the concept of the power-law memory. We numerically demonstrate the stability of the fractional evolutionary equilibrium. Extensive numerical results have been conducted to evaluate the performance of the fractional evolutionary game.**

***Index Terms*—Network selection, fractional evolutionary game, memory-affecting rationality, and heterogeneous network.**

I. Introduction

Due to the proliferation of wireless handsets and portable devices as well as data-hungry multimedia applications, mobile data demand continues to grow exponentially in recent years and is likely to soon outgrow the capacity of the current cellular networks. To address this severe issue, the network will continue to become increasingly heterogeneous as we move to fifth generation (5G) [1]. By taking advantage of the best of different networking technologies, multi-faceted benefits can be reaped in 5G heterogeneous networks, such as improving the utilization of the network resources, enhancing the scalability and provisioning networking service upon requirement [2].

In this paper, we study a user-centric network selection problem in 5G HetNets, where the users can freely select the network and access the networking service of which. Moreover, we study the network selection problem on a long-run basis such that the behaviors of the users can be investigated dynamically. It is worth noting here that to study such a long-run network selection problem, it is natural and practical to consider that the users' memory, i.e., past service experience, will affect their decision-making. In other words, the users make their decisions by taking into account not only their instantaneous achievable service experience but also their past service experience. In brief, the action that the user selects the network can be regarded as the behavior of an economic agent in an economic process. In the economic process, the economic agent is aware of and pays great attention to the history of this process, hence the impact of which on the behavior cannot be ignored. For example, one recent study reported that the enrollment rates at a given college were dramatically influenced by whether the students happened to visit that campus on a sunny day instead of a rainy day [3].

Nevertheless, the conventional dynamical model, i.e., classical evolutionary game, cannot capture the impact of the users' memory on their decision-making due to the fact that the players in the classical evolutionary game only consider the instantaneous achievable utility. In this case, we incorporate the concept of the power-law memory [4], which is used to depict the impact of the users' memory on their strategies. That is, whenever the users are making decisions on their current strategies, they will take into consideration not only their instantaneous achievable utility but also their previous decisions within the memory [5].

In this paper, we study a dynamic network selection problem in 5G HetNets as shown in Fig. 1, where a HetNet constituted of ultra-high frequency (UHF), i.e., the frequencies below 6 GHz, base station (BS), millimeter-wave (mmWave) BS, and unmanned aerial vehicle (UAV)-enabled mmWave BS is considered to be the application scenario for the 5G HetNet. In the problem, there exists two parties, i.e., different types of BSs working as the utility providers with a flat-rate pricing scheme for provisioning the communication service and the communication service customers, i.e., memory-affecting rational users. Specifically, we first formulate a classical evolutionary game to analyze the interaction among the users in the 5G HetNet on a long-run basis. Then, by incorporating the concept of the power-law memory, which is depicted by using the fractional calculus (including fractional derivatives and integrals), we cast the classical evolutionary game as a fractional evolutionary game to investigate the dynamic behaviors of the memory-affecting rational users.

II. Preliminary

As shown in Fig. 1, we consider a 5G HetNet constituted of three types of BSs, i.e., UHF BS, mmWave BS, and UAV-enabled mmWave BS, as the application scenario. Here, the propagation models of the aforementioned types of BSs are presented in Section II-A. For simplicity, we assume that ideal backhaul links exist between the UAV-enabled mmWave BSs and their nearby mmWave BSs such that the throughput of the backhauls of the UAV-enabled mmWave BSs will not affect

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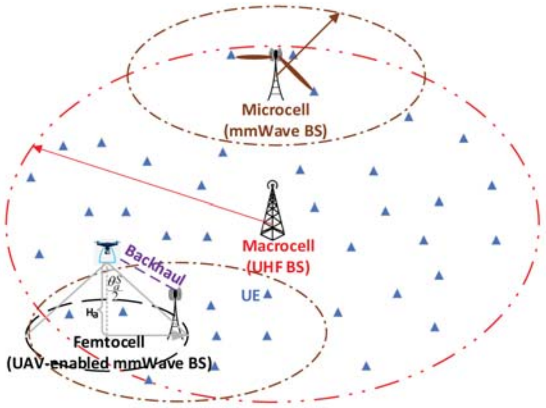

Figure 1: 5G Heterogeneous Network Architecture

the throughput of their downlinks. Note here that the user is considered to have an omnidirectional antenna.

Additionally, to incorporate the concept of power-law memory, we introduce the left-sided Caputo fractional derivative with respect to time [5] in Section II-B. The power-law memory can be captured by the left-sided Caputo fractional derivative [5].

A. Network Model

1) UHF Propagation Model: A set of UHF BSs, denoted by $\mathcal{U}$, is deployed to provide UHF communication service. The received downlink signal power at user $i \in \mathcal{N}$ from UHF BS $u \in \mathcal{U}$ is $P_{u,i} = P_u^{\mathrm{t}} h_{u,i} \rho_u G_u \left[L_u\left(l_u - l_i\right)\right]^{-1}$, where l_u, $l_i \in \mathbb{R}^{3\times1}$ respectively denote the locations of UHF BS u and user i[1], $L_u\left(z\right) = \|z\|^{\alpha_u}$ is the path-loss function, $h_{u,i}$ is the small-scale fading, G_u is the antenna gain, ρ_u is the near-field path loss at 1 m, i.e., $\rho_u = \left(\frac{c}{4\pi f_u}\right)^2$, c represents the speed of light, f_u is the carrier frequency, and P_u^{t} is the transmit power. Let $\mathcal{I}_u$ denote the set of the UHF BSs using the same channel as that of UHF BS u, the co-channel interference of UHF BS u is $\sum\limits_{w\in\mathcal{I}_u} P_w^{\mathrm{t}} h_{w,i}\rho_w G_w \left[L_w\left(l_w - l_i\right)\right]^{-1}$, and the SINR for user i at UHF BS u is $\frac{P_u^{\mathrm{t}} h_{u,i}\rho_u G_u [L_u(l_u-l_i)]^{-1}}{\sum\limits_{w\in\mathcal{I}_u} P_w^{\mathrm{t}} h_{w,i}\rho_w G_w [L_w(l_w-l_i)]^{-1}+\sigma_u^2}$, where σ_u^2 is noise power. The corresponding downlink transmission rate is

$$\begin{aligned}R_{u,i} &= \frac{W_u}{N_u}\log_2\left(1+\frac{P_{u,i}}{\sum\limits_{w\in\mathcal{I}_u} P_{w,i}+\sigma_u^2}\right)\\ &= \frac{W_u}{N_u}\log_2\left(1+\frac{P_u^{\mathrm{t}} h_{u,i}\rho_u G_u\left[L_u\left(l_u-l_i\right)\right]^{-1}}{\sum\limits_{w\in\mathcal{I}_u} P_w^{\mathrm{t}} h_{w,i}\rho_w G_w\left[L_w\left(l_w-l_i\right)\right]^{-1}+\sigma_u^2}\right),\end{aligned} \tag{1}$$

where W_u is the bandwidth of UHF channel at UHF BS u and N_u is the number of users selecting UHF BS u. Here, we consider that UHF BSs use TDMA, and hence the number of users selecting UHF BS $w \in \mathcal{I}_u$ will not affect the co-channel interference received by the users of UHF BS u. Note here that $R_{u,i}$ represents the per-user transmission rate averaged over frame time.

2) mmWave Propagation Model: The set of mmWave BSs is denoted by $\mathcal{M}$. The received signal power at user i in the downlink from an mmWave BS $m \in \mathcal{M}$ is

$$P_{m,i} = p_{m,i}^{\mathrm{LOS}} P_m^{\mathrm{t}} h_{m,i}\rho_m G_{m,i}\left[L_m\left(l_m - l_i\right)\right]^{-1}, \tag{2}$$

where $l_m \in \mathbb{R}^{3\times1}$ is the location of mmWave BS m[2], $L_m\left(z\right) = \|z\|^{\alpha_m^s}$ is the path-loss function, therein $s \in \{\mathrm{LOS}, \mathrm{NLOS}\}$ is the link indicator, $h_{m,i}$ is the small-scale fading, $G_{m,i}$ is the antenna gain, ρ_m is the near-field path loss at 1m, i.e., $\rho_m = \left(\frac{c}{4\pi f_m}\right)^2$, and P_m^{t} is the transmit power. $p_{m,i}^{\mathrm{LOS}}$ is the line-of-sight probability function, i.e., $r_{m,i} = \|l_m - l_i\|$, given by [6]

$$p_{m,i}^{\mathrm{LOS}} = \begin{cases} C, & \text{if } r_{m,i} \le D,\\ 0, & \text{otherwise,}\end{cases} \tag{3}$$

where $C \in [0, 1]$ can be interpreted as the average LOS area in the spherical region around a typical user, e.g., $[C, D] = [0.081, 250]$ for Chicago [7]. The mmWave BSs are equipped with directional antennas and the antenna gain of mmWave BS m for user i, i.e., $G_{m,i}$, is given by

$$G_{m,i}\left(\theta_{m,i}\right) = \begin{cases} G^{\mathrm{M}}, & \text{if } |\theta_{m,i}| \le \frac{\theta_m^{\mathrm{S}}}{2},\\ G^{\mathrm{S}}, & \text{otherwise,}\end{cases} \tag{4}$$

where $\theta_{m,i}$ is the user i's angle with respect to the best beam alignment, and θ_m^{S} represents the main beamwidth of mmWave BS m. The SNR for user i at mmWave BS m is $p_{m,i}^{\mathrm{LOS}} P_m^{\mathrm{t}} h_{m,i}\rho_m G_{m,i}\left[L_m\left(l_m - l_i\right)\right]^{-1}/\sigma_m^2$, where σ_m^2 is the noise power in the mmWave BS m's channel. Correspondingly, the downlink transmission rate of user i at mmWave BS m is

$$\begin{aligned}R_{m,i} &= \frac{W_m}{N_m}\log_2\left(1+\frac{P_{m,i}}{\sigma_m^2}\right)\\ &= \frac{W_m}{N_m}\log_2\left(1+\frac{p_{m,i}^{\mathrm{LOS}} P_m^{\mathrm{t}} h_{m,i}\rho_m G_{m,i}\left[L_m\left(l_m-l_i\right)\right]^{-1}}{\sigma_m^2}\right),\end{aligned} \tag{5}$$

where W_m is the bandwidth at mmWave BS m and N_m is the number of users selecting mmWave BS m.

3) UAV-enabled mmWave Propagation Model: The set of UAV-enabled mmWave BSs is denoted by $\mathcal{A}$. The received signal power at user i in the downlink from an UAV-enabled mmWave BS $a \in \mathcal{A}$ is $P_{a,i} = p_{a,i}^{\mathrm{LOS}} P_a^{\mathrm{t}} h_{a,i}\rho_a G_{a,i}\left[L_a\left(l_a - l_i\right)\right]^{-1}$, where the definitions of P_a^{t}, $h_{a,i}$, ρ_a, and $L_a\left(z\right)$ are similar to that in (2). Here, $l_a \in \mathbb{R}^{3\times1}$ is the location of UAV-enabled mmWave BS a, the third component of which, denoted by H_a, indicates the altitude. Similar to [8], each UAV-enabled mmWave BS $a \in \mathcal{A}$ is equipped with a directional antenna pointing downward at the ground, whose half-power beamwidths are θ_a^S radians with

[1]The third components of the three-dimensional vectors l_u and l_i indicate the altitudes of UHF BS u and user i, respectively.

[2]The third component of l_m indicates the altitude of mmWave BS m.

$\frac{\theta_a^S}{2} \in \left(0, \frac{\pi}{2}\right)$. Then, the corresponding antenna gain for user i at UAV-enabled mmWave BS a is [9]

$$G_{a,i}\left(l_a, l_i\right) = \begin{cases} G^A, & \text{if } \left\| l_a|_{H_a=0} - l_i \right\| \leq H_a \tan \frac{\theta_a^S}{2}, \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

$p_{a,i}^{\mathrm{LOS}}$ is the line-of-sight probability as a function of the distance between the user i and UAV-enabled mmWave BS a on the 2-D plane, i.e., $r_{a,i} = \left\| l_a|_{H_a=0} - l_i \right\|$, as well as the altitude of UAV-enabled mmWave BS a, i.e., H_a, given by $p_{a,i}^{\mathrm{LOS}}\left(r_{a,i}, H_a\right) = \frac{1}{1+b\exp\left(-c\left(\frac{180}{\pi}\tan^{-1}\left(\frac{H_a}{r_{a,i}}\right)-b\right)\right)}$, where b and c are the constants depending on the environment, and $l_a|_{H_a=0}$ is the projection of UAV-enabled mmWave BS a on the ground [10]. Due to the directional antenna equipped by the UAV-enabled mmWave BSs, the corresponding expected SNR of user i at UAV-enabled mmWave BS a is $\frac{p_{a,i}^{\mathrm{LOS}} P_a^{\mathrm{t}} h_{a,i} \rho_a G_{a,i} [L_a(l_a - l_i)]^{-1}}{\sigma_a^2}$, where σ_a^2 is the noise power. The corresponding downlink transmission rate of user i at UAV-enabled mmWave BS a is

$$\begin{aligned} R_{a,i} &= \frac{W_a}{N_a}\log_2\left(1 + \frac{P_{a,i}}{\sigma_a^2}\right) \\ &= \frac{W_a}{N_a}\log_2\left(1 + \frac{p_{a,i}^{\mathrm{LOS}} P_a^{\mathrm{t}} h_{a,i}\rho_a G_{a,i}\left[L_a\left(l_a - l_i\right)\right]^{-1}}{\sigma_a^2}\right), \end{aligned} \quad (7)$$

where W_a and N_a are the bandwidth at and the number of users selecting UAV-enabled mmWave BS a, respectively.

B. The Concept of the Power-law Memory

As introduced in [4], most of the economic process are memory-aware. To study the memory-aware economic processes, fractional calculus have been regarded as a promising approach to investigate the role of the memory in the economic processes such as [11]. Specifically, the memory-aware economic processes are derived as follows:

1) Basically, the most general formulation of the economic processes can be presented as $Y(t) = F_0^t(X(\tau)) + Y_0$, where F_0^t denotes a mapping that can transform the history of the changes of $X(\tau)$, $\forall \tau \in [0, t]$ so as to find the value of $Y(t)$, $X(\tau)$ and $Y(t)$ are the time-dependent input and output, respectively, and $Y_0 = Y(0)$ is the initial state of the output. The integer-order integral based approach, i.e., $F_0^t(X(\tau)) := \int\limits_0^t X(\tau)\,\mathrm{d}\tau$, has been widely adopted to study the dynamics in such economic processes, e.g., [12].
2) However, a vital drawback exists in the integer-order integral based approach. By taking derivative of $Y(t) = \int\limits_0^t X(\tau)\,\mathrm{d}\tau + Y_0$ with respect to t, the economic processes based on the integer-order integral can be represented as $\frac{\mathrm{d}}{\mathrm{d}t}Y(t) = X(t)$. Here, $\frac{\mathrm{d}}{\mathrm{d}t}Y(t)$ does not have any information about $X(\tau)$, $\forall \tau \in [0, t)$, which means that the reaction of the economic processes based on the integer-order integral is not aware of the changes of $X(\tau)$, $\forall \tau \in [0, t)$ and further not memory-aware.
3) To address this drawback, the economists in [4] considered the Volterra operator, i.e., a mapping, which can be specifically expressed as $F_0^t(X(\tau)) := \int\limits_0^t M_\beta(t-\tau) X(\tau)\,\mathrm{d}\tau$, where $M_\beta(t-\tau)$ is a weighting function to measure the impact level of the previous input $X(\tau)$ on the current output $Y(t)$ according to the time distance between τ and t. Additionally, by introducing such a weighting function into the economic process, the first derivative of $Y(t)$, i.e., $\frac{\mathrm{d}}{\mathrm{d}t}Y(t) = M_\beta(t) X(0) + \int_0^t M_\beta(t-\tau)\left[\frac{\mathrm{d}}{\mathrm{d}\tau}X(\tau)\right]\mathrm{d}\tau$, no longer depends only on $X(t)$ but also on $X(\tau)$ with $\tau \in [0, t)$. The specifical form of $M_\beta(t-\tau)$ is considered to be in the power form, i.e., $M_\beta(t-\tau) = \frac{1}{\Gamma(\beta)}\frac{1}{(t-\tau)^{1-\beta}}$, where
$$\Gamma(z) = \int_0^{+\infty} x^{z-1} e^{-x}\,\mathrm{d}x \quad (8)$$
is the gamma function [13].
4) To represent the economic processes in the expression of fractional equation, we take the derivation of $Y(t)$ at the order of β by using the left-sided Caputo fractional derivative. Hence, the memory-aware economic processes can be represented as ${}_0^{\mathrm{C}}D_t^\beta Y(t) = X(t)$ with $Y(0) = Y_0$, where ${}_0^{\mathrm{C}}D_t^\beta Y(t)$ is the left-sided Caputo fractional derivative of $Y(t)$ at the order of β defined as follows [5]
$${}_0^{\mathrm{C}}D_t^\beta Y(t) = \frac{1}{\Gamma(\lceil\beta\rceil - \beta)}\int_0^t \frac{Y^{(\lceil\beta\rceil)}(\tau)}{(t-\tau)^{\beta+1-\lceil\beta\rceil}}\mathrm{d}\tau, \quad (9)$$
and $\lceil\beta\rceil$ denotes the integer obtained by rounding up β.

III. System Description

In this section, we consider a HetNet constituted of UHF BS u, mmWave BS m, and UAV-enabled mmWave BS a. The homogeneous users are regarded as a group, denoted by $\mathcal{N}^G$, to select the BSs and access the communication services. That is, N_u^G, N_m^G, and N_a^G users will select UHF BS u, mmWave BS m, and UAV-enabled mmWave BS a, respectively, where $N_u^G + N_m^G + N_a^G = N^G$. Accordingly, each user in the group selects UHF BS u, mmWave BS m, and UAV-enabled mmWave BS a at the probabilities of $y_u = \frac{N_u^G}{N^G}$, $y_m = \frac{N_m^G}{N^G}$, and $y_a = \frac{N_a^G}{N^G}$, respectively. Here, in the system model of the HetNet with homogeneous users, we assume that the users are homogeneous in the long-term basis, e.g., indoor users working in the office.

1) User's Utility: Based on the expression of the downlink transmission rate of UHF BS u, i.e., (1), the users selecting UHF BS u will share the bandwidth of UHF BS u. As the expected number of the users selecting UHF BS u is $y_u N^G$, the user can obtain the expected bandwidth of $\frac{W_u}{y_u N^G}$ every time slot and the expected downlink transmission rate of

$$\tilde{R}_u = \frac{W_u}{y_u N^G}\log_2\left(1 + \frac{P_u}{\sigma_u^2}\right). \quad (10)$$

Let λ_u denote the intrinsic value of unit bitrate and ϕ_u be the price of downloading one unit data through UHF BS u's communication service, the utility including the cost that each user can obtain by selecting UHF BS u is given by

$$\Pi_u = \lambda_u \tilde{R}_u - \phi_u \tilde{R}_u = \left(\lambda_u - \phi_u\right) \tilde{R}_u = \lambda_u^{\phi} \tilde{R}_u, \tag{11}$$

where $\lambda_u^{\phi} = \lambda_u - \phi_u$.

Similar to (10) and based on (5) and (7), the expected downlink transmission rates of mmWave BS m and UAV-enabled mmWave BS a for each user are $\tilde{R}_m = \frac{W_m}{y_m N^G} \log_2 \left(1 + \frac{P_m}{\sigma_m^2}\right)$ and $\tilde{R}_a = \frac{W_a}{y_a N^G} \log_2 \left(1 + \frac{P_a}{\sigma_a^2}\right)$, respectively. Then, the utilities of each user obtained from selecting mmWave BS m and UAV-enabled mmWave BS a are respectively

$$\Pi_m = \lambda_m \tilde{R}_m - \phi_m \tilde{R}_m = \left(\lambda_m - \phi_m\right) \tilde{R}_m = \lambda_m^{\phi} \tilde{R}_m \tag{12}$$

and

$$\Pi_a = \lambda_a \tilde{R}_a - \phi_a \tilde{R}_a = \left(\lambda_a - \phi_a\right) \tilde{R}_a = \lambda_a^{\phi} \tilde{R}_a, \tag{13}$$

where λ_m and λ_a are respectively the intrinsic values of unit bitrate for mmWave BS m and UAV-enabled mmWave BS a, ϕ_m and ϕ_a are respectively the price of downloading one unit data through the mmWave BS m's and UAV-enabled mmWave a's communication service, $\lambda_m^{\phi} = \lambda_m - \phi_m$, and $\lambda_a^{\phi} = \lambda_a - \phi_a$.

2) Game Formulation for the Heterogeneous Cellular Network with Homogeneous Users: With the utility functions defined in (11), (12), and (13), i.e., Π_u, Π_m, and Π_a, respectively, the average utility of the user is $\bar{\Pi} = y_u \Pi_u + y_m \Pi_m + y_a \Pi_a$. Then, the replicator dynamics yields the following classical evolutionary game:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} y_u(t) &= \exp(-\delta)\, y_u(t) \left[\Pi_u(t) - \bar{\Pi}(t)\right], \qquad y_u(0) = y_u^0, \\ \frac{\mathrm{d}}{\mathrm{d}t} y_m(t) &= \exp(-\delta)\, y_m(t) \left[\Pi_m(t) - \bar{\Pi}(t)\right], \qquad y_m(0) = y_m^0, \\ \frac{\mathrm{d}}{\mathrm{d}t} y_a(t) &= \exp(-\delta)\, y_a(t) \left[\Pi_a(t) - \bar{\Pi}(t)\right], \qquad y_a(0) = y_a^0. \end{aligned} \tag{14}$$

Moreover, by incorporating the power-law memory, we formulate the fractional evolutionary game based on the left-sided Caputo fractional derivative as follows:

$$\begin{aligned} {}_0^{\mathrm{C}}\mathrm{D}_t^{\beta} y_u(t) &= \exp(-\delta)\, y_u(t) \left[\Pi_u(t) - \bar{\Pi}(t)\right], \qquad y_u(0) = y_u^0, \\ {}_0^{\mathrm{C}}\mathrm{D}_t^{\beta} y_m(t) &= \exp(-\delta)\, y_m(t) \left[\Pi_m(t) - \bar{\Pi}(t)\right], \qquad y_m(0) = y_m^0, \\ {}_0^{\mathrm{C}}\mathrm{D}_t^{\beta} y_a(t) &= \exp(-\delta)\, y_a(t) \left[\Pi_a(t) - \bar{\Pi}(t)\right], \qquad y_a(0) = y_a^0. \end{aligned} \tag{15}$$

IV. Equilibrium Analysis

In this section, we analyze the equilibrium of the fractional evolutionary game defined in (15). Considering the fractional evolutionary game defined in (15), let $\mathbf{X}(t) = \left[x_{w,i}(t)\right]_{w \in \mathcal{U} \cup \mathcal{M} \cup \mathcal{A}, i \in \mathcal{N}}$ and $\mathbf{F}(\mathbf{X}(t)) = \left[\exp(-\delta)\, x_{w,i}(t) \left[U_{w,i}(t) - \bar{U}_i(t)\right]\right]_{w \in \mathcal{U} \cup \mathcal{M} \cup \mathcal{A}, i \in \mathcal{N}}$, we accordingly have the fractional evolutionary game (15) in a simplified form as follows:

$${}_0^{\mathrm{C}}\mathrm{D}_t^{\beta} \mathbf{X}(t) = \mathbf{F}(\mathbf{X}(t)), \quad \forall t \in \mathcal{T} = [0, T], \tag{16}$$

where $\mathbf{X}(0) = \mathbf{X}^0 = \left[x_{w,i}^0\right]_{w \in \mathcal{U} \cup \mathcal{M} \cup \mathcal{A}, i \in \mathcal{N}}$.

Theorem 1. *If f_j, i.e., the j-th element of the vector $\mathbf{F}$, $j \in \{1, \ldots, N(U + A + M)\}$, satisfies the following conditions:*

- *$f_j : \mathcal{D} \mapsto \mathbb{R}^+$ is second-order continuous,*
- *$\frac{\partial f_j}{\partial x_{w,i}}$ exists and is bounded, $\forall w \in \mathcal{U} \cup \mathcal{M} \cup \mathcal{A}$, $\forall i \in \mathcal{N}$,*

(16) has an equivalent problem as follows:

$$\mathbf{X}(t) = \mathbf{X}^0 + {}_0\mathrm{I}_t^{\beta} \mathbf{F}(\mathbf{X}(t)), \quad \forall t \in \mathcal{T} = [0, T], \tag{17}$$

where ${}_0\mathrm{I}_t^{\beta}$ is the fractional integral, i.e., ${}_0\mathrm{I}_t^{\beta} f(t) = \int_0^t \frac{(t-\tau)^{\beta-1}}{\Gamma(\beta)} f(\tau)\, \mathrm{d}\tau$ [14], the second condition implies the Lipschitz condition, i.e., for all $j \in \{1, 2, \ldots, N(U + A + M)\}$, $\left|f_j(\mathbf{X}(t)) - f_j(\mathbf{Y}(t))\right| < L \left\|\mathbf{X}(t) - \mathbf{Y}(t)\right\|_{\mathcal{L}_1}$, where $L \in \mathbb{R}^+$, and N, U, A, and M are the cardinalities of $\mathcal{N}$, $\mathcal{U}$, $\mathcal{A}$, and $\mathcal{M}$, respectively.

Proof. Due to the limited space, we omit the proof here. □

Then, we prove that the equivalent problem defined in (17) admits a unique solution, which implies that (16) also admits a unique solution.

Theorem 2. *With the conditions presented in Theorem 1, the problem defined in (17) admits a unique solution.*

Proof. Due to the limited space, we omit the proof here. □

At last, we investigate the stability, i.e., robustness, of the fractional evolutionary game defined in (16) in Theorem 3 through studying the variations incurred by the small perturbations of the boundary condition.

Theorem 3. *With conditions presented in Theorem 1, the equilibrium of the fractional evolutionary game defined in (16) is uniformly stable.*

Proof. Due to the limited space, we omit the proof here. □

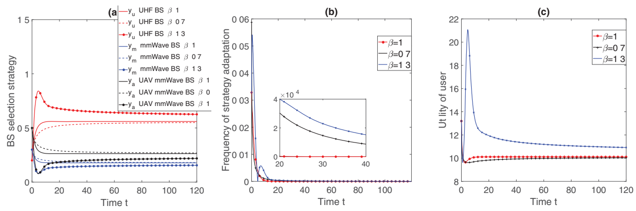

Figure 2: (a) The user's network selection strategy, (b) frequency of strategy adaptation, and (c) the user's utility in the classical and fractional evolutionary games

V. Performance Evaluation

The parameter setting for the network model of the HetNet is shown in Table I. We first compare the results obtained from the fractional evolutionary games with $\beta = 0.7$ and $\beta = 1.3$ with that obtained from the classical evolutionary game, i.e., $\beta = 1$. As shown in Fig. 2(a), the user's strategy in the fractional evolutionary game with $\beta = 1.3$ evolves more rapidly to the vicinity of the equilibrium strategy than that in the classical evolutionary game and the fractional evolutionary game with $\beta = 0.7$. This means that the strategy adaptation rate of the users in the fractional evolutionary game with $\beta = 1.3$ is faster than that in both the classical evolutionary game and the fractional evolutionary game with $\beta = 0.7$. However, it can be observed in Fig. 2(b) that the user's

Table I: Parameters for the Network Model

Symbol	Typical Value	Symbol	Typical Value	Symbol	Typical Value
W_m, W_u W_a	1 GHz, 20 MHz, 1 GHz	α_m^{LOS}, α_m^{NLOS}, α_u	2, 4, 2.7	$h_{u,i}$, $h_{m,i}$, $h_{a,i}$	exp (1)
P_u, P_m, P_a	46 dBm, 30 dBm, 23 dBm	G_u, $G_{m,i}$, $G_{a,i}$	0 dBi, (G^M, G^S), G^A	H_a	20m
σ_u^2, σ_m^2, σ_a^2	-174dBm/Hz $+ 10\log_{10}(W_u) +$ 10dB	G^M, G^S, G^A	18 dBi, -2 dBi, 18 dBi	b, c	1.5, 1
f_m, f_u	70 GHz, 1.8 GHz (e.g., 4G/LTE)	C, D	0.081, 250 m	θ_a^S	45°

Table II: Location of the BSs and user group

BSs and user	Coordinate (Km)	Parameters	Value
UHF BS	[1 0 0]	λ_u^ϕ	10^{-7}
mmWave BS	[0 0 0]	λ_m^ϕ	1.5×10^{-9}
UAV-enabled mmWave BS	[0 0.1 0.02]	λ_a^ϕ	10^{-9}
User group	[0 0.1 0]	$\mathcal{N}^G$, δ	10, 2

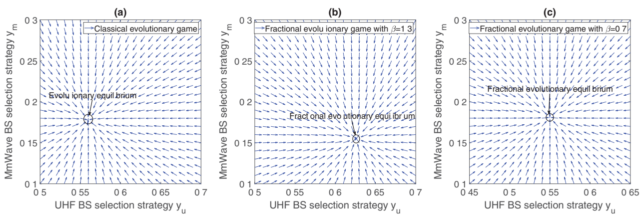

Figure 3: Direction field of the replicator dynamics for verifying the stability of the strategies in the (fractional) evolutionary equilibrium when $t = 120$

strategies in the fractional evolutionary game with $\beta = 1.3$ converge to the equilibrium more slowly than that in both the classical evolutionary game and the fractional evolutionary game with $\beta = 0.7$. The reason is that the reaction speed of the user in the classical evolutionary game is faster than that in the fractional evolutionary game with $\beta = 0.7$ while slower than that in the fractional evolutionary game with $\beta = 1.3$.

In addition, we also evaluate the user's utility under different memory effects, i.e., different values of β. As shown in Fig. 2(c), the memory effect with $\beta = 0.7 \in (0, 1)$ can lead to a worse utility for the user compared with the model without memory effect, i.e., $\beta = 1$. In contrast, the memory effect with $\beta = 1.3 \in (1, 2)$ can lead to a better utility for the user compared with the model without memory effect. In this case, we would like to define the memory effect with $\beta \in (0, 1)$ as negative memory effect and that with $\beta \in (1, 2)$ as positive memory effect.

To verify the stability of the strategies in the classical and fractional evolutionary games, we present the direction field of the replicator dynamics. As shown in Fig. 3, any unstable strategy will follow the arrow to reach the equilibrium strategy, which is marked by the black circle. This demonstrates the convergence and stability of the strategy and is consistent with Theorem 3.

VI. Conclusion

We have presented dynamic game framework to analyze the strategies of the users in the HetNets. The interaction among the users has been modeled as a fractional evolutionary game, where the dynamic network selection strategies of the memory-affecting rational users are captured by the replicator dynamics and the concept of the power-law memory. We numerically verified the stability of the fractional evolutionary equilibrium. The stable and unique fractional evolutionary equilibrium has been obtained as the solution to the fractional evolutionary game. Moreover, we have presented a series of insightful analytical and numerical results on the equilibrium of the fractional evolutionary games.

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